Peace Dividends and Economic Inequality Expansion: A General Equilibrium Simulation- General Case

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February, 2012
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Introduction

Prominent economists such as Krugman [2007] and Summers [2011] argued that income distribution inequality must be reduced in the US. The expansion of income distribution inequality, however, has been noticed worldwide. They pointed out economic reasons such as technological innovation and globalism for the expanded inequality, without elaborating the analysis. Unemployment due to these factors may be regarded by them as the underlying mechanism. It may be of some interest, then, to examine whether the expansion emerges if full employment is guaranteed when a country experiences innovation or globalization. From the viewpoint of globalism, Fukiharu [2011a] examined the expansion of income distribution inequality utilizing the Heckscher-Ohlin trade model. Meanwhile, from the viewpoint of innovation, Fukiharu [2011b] examined the inequality expansion utilizing traditional general equilibrium model when a new commodity is invented and used as a private good in a country. The present paper follows the examination by Fukiharu [2011b] and examines how the inequality expansion differs when the new commodity is a public good. This examination is also of some interest theoretically especially when we consider the economic history in the “peace dividends” era. After the collapse of the communist countries in the 1980s, the US president Bill Clinton could shift the government fund from the use for military purpose to the non-military purpose. It is pointed out that he did shift the government fund to the promotion of ICTs, succeeding in the emergence of “new economy” in the 1990s. It is interesting, in this way, to examine how the inequality expansion differs when the government fund was used for the promotion of innovating military good and a new military good was invented and used as a public good.

I: Model, Assumptions, and Simulation Procedure

Fukiharu [2011b] starts with the traditional general equilibrium model under the “decreasing-returns-to-scale” production functions for a country A with two sectors.
The production functions for country A regarding the two industries are assumed to be of the same Cobb-Douglas type, where the one for the first sector is \( f_1A = L_1^{a_1}K_1^{b_1} \), the one for the second sector is \( f_2A = L_2^{a_2}K_2^{b_2} \) where \( a_1+b_1<1 \) (i=1, 2). It is assumed that country A is endowed with \( LcA \) units of Labor and \( KcA \) units of capital. Specifying parameters, first, Fukiharu [2011b] computes general equilibrium, “basic GE”, for country A in national isolation with Gini coefficient for the “basic GE” income distribution. Next, he modifies the economy in country A, supposing that a new commodity is invented with “decreasing-returns-to-scale” production function

\[ f_3A = L_3^{a_3}K_3^{b_3} \]

where \( a_3+b_3<1 \) and used as a private good in country A. He computes the Gini coefficient for the country A’s “modified GE” income distribution, examining whether income distribution becomes more unequal or not by this innovation.

In the present paper, we repeat the computation of country A’s “modified GE” income distribution in the following subsection I-1. In the next subsection I.2, we proceed to the computation of “modified GE” when the new commodity is a public good and procured by the Lindahl mechanism.

### I.1: The Invention of New Commodity for a Country A: Private Good Case and Gini Coefficient: \( \text{gini}_{A0} \)

We start with the computation of “modified GE” for a country A under national isolation, when the third commodity is invented and in use as a private good. The assumptions and procedure are exactly the same as in Fukiharu [2011, chapter I].

#### Production of country A

Country A is under national isolation. She has three sectors of production, which produces different consumption good, utilizing labor, \( L_i \), and capital, \( K_i \), \( y \) stands for the output of sector 1, \( x \) stands for the output of sector 2, and \( z \) stands for the output of sector 3. (i=1, 2, 3) Production function of sector 1, \( y = f_1A \), is assumed with \( a_1+b_1<1 \): decreasing returns to scale, with exactly the same values as in the “basic GE” case. Production function of sector 2, \( x = f_2A \), is assumed, with \( a_2+b_2<1 \): decreasing returns to scale, with exactly the same values as in the “basic GE” case. Production function of sector 3, \( z = f_3A \), is assumed with \( a_3+b_3<1 \): decreasing returns to scale.

From the profit maximization of the sector 1, demand for labor, \( L_1A^D \), and demand for capital, \( K_1A^D \), are derived, where \( p_y \) stands for the price of the consumption good, \( y \), exactly the same as in the “basic GE” case. Thus, supply function of \( y \), \( y_A^S \), is derived, with \( p_y \), \( w_L \), and \( w_K \), as parameters, exactly the same as in the “basic GE” case. Profit function of sector 1, \( \pi_1A \), is computed, with \( p_y \), \( w_L \), and \( w_K \), as parameters. This profit accrues to entrepreneur 1.
From the profit maximization of the sector 2, demand for labor, $L_2A^D$, and demand for capital, $K_2A^D$, are derived, where $p_x$ stands for the price of the consumption good, $x$, exactly the same as in the “basic GE” case. Thus, supply function of $x, x_A^S$, is derived, with $p_x, w_L$, and $w_K$, as parameters, exactly the same as in the “basic GE” case. Profit function of sector 2, $\pi_2A$, is derived, with $p_x, w_L$, and $w_K$, as parameters. This profit accrues to entrepreneur 2.

From the profit maximization of the sector 3, demand for labor, $L_3A^D$, and demand for capital, $K_3A^D$, are derived, where $p_z$ stands for the price of the consumption good, $z$. Thus, supply function of $z, z_A^S$, is derived, with $p_z, w_L$, and $w_K$, as parameters. Profit function of sector 3, $\pi_3A$, is computed as follows, with $p_z, w_L$, and $w_K$, as parameters. This profit accrues to entrepreneur 3.

### Consumption of country A

We proceed to the demand side of country A. She is endowed with $L_{cA}$, initial endowment of labor, and $K_{cA}$, initial endowment of capital, exactly the same as in the “basic GE” case.

All the agents in this paper: workers, capitalists, and entrepreneurs, have the same CES type utility function, $u[y, x, z] = (\gamma^a + x^a + z^a)^{1/a}$, $a<1$.

All the consumers maximize utility subject to income constraint:

$$\max u[y, x, z] \quad \text{s.t.} \quad p_x y + p_x x + p_z z = m \quad (1)$$

where $m$ is income. Worker's income consists of initial endowment of labor, evaluated by the wage rate: $w_L L_{cA}$. It is assumed that they supply $L_{cA}$ for labor supply. It is assumed that they supply $L_{cA}$ for labor supply. Capitalist's income consists of initial endowment of capital, evaluated by the rental price of capital: $w_K K_{cA}$. It is assumed that they supply $K_{cA}$ for capital supply. Entrepreneur 1’s income consists of profit for the sector 1, $\pi_1A$. Entrepreneur 2’s income consists of profit for the sector 2, $\pi_2A$. Finally, entrepreneur 3’s income consists of profit for the sector 3, $\pi_3A$.

Demand function of workers for commodity $y$, $y_L^D$, that for commodity $x$, $x_L^D$, that for commodity $z$, $z_L^D$, demand function of capitalists for commodity $y$, $y_K^D$, that for commodity $x$, $x_K^D$, that for commodity $z$, $z_K^D$, demand function of entrepreneur 1 for commodity $y$, $y_{E1}^D$, that for commodity $x$, $x_{E1}^D$, that for commodity $z$, $z_{E1}^D$, and demand function of entrepreneur 2 for commodity $y$, $y_{E2}^D$, that for commodity $x$, $x_{E2}^D$, that for commodity $z$, $z_{E2}^D$, are derived.

Country A’s demand for commodity $y$, $y_A^D$, is the sum of $y_L^D, y_K^D, y_{E1}^D, y_{E2}^D$, and $y_{E3}^D$. Country A’s demand for commodity $x$, $x_A^D$, is the sum of $x_L^D, x_K^D,$
$x_{E1}^D, x_{E2}^D, x_{E3}^D$. Country A’s demand for commodity $z$, $z_A^D$, is the sum of $z_L^D$, $z_K^D$, $z_{E1}^D$, $z_{E2}^D$, $z_{E3}^D$.

“Modified GE” and Private Good Gini Coefficient, gini$_{A0}$

General equilibrium for country A with innovation, “modified GE”, $\{p_y^*, p_x^*, p_z^*, w_k^*, w_L^*\}$, is defined by

\[
\begin{align*}
y_A^D &= y_A^S, \quad (2) \\
x_A^D &= x_A^S, \quad (3) \\
z_A^D &= z_A^S, \quad (4) \\
L_{1A}^D + L_{2A}^D + L_{3A}^D &= L_{cA}, \quad (5) \\
K_{1A}^D + K_{2A}^D + K_{3A}^D &= K_{cA}. \quad (6)
\end{align*}
\]

As is well known, by the Walras law, 4 conditions, (2)~(6), are not independent. Thus, assuming $w_L = 1$, we compute “modified GE” prices: $p_y^*$, $p_x^*$, $p_z^*$, and $w_k^*$, as a solution to (2)~(5).

It is ascertained that these prices are indeed “modified GE”, by showing that $\{p_y^*, p_x^*, p_z^*, w_k^*\}$ is a solution to (2), (3), (4), and (6). From the “modified GE” prices the “modified GE” incomes for 5 economic agents: (aggregate) workers, (aggregate) capitalists, entrepreneur 1, entrepreneur 2, and entrepreneur 3, are derived. Utilizing the “modified GE” incomes for 5 agents, “modified GE” Gini coefficient, gini$_{A0}$, is computed.

I.2 : The Invention of New Commodity for a Country A: Public Good Case and Gini Coefficient Before Tax, gini$_{11}$, and Gini Coefficient After Tax, gini$_{12}$

We start with the computation of “modified GE with public good” for a country A under national isolation, when the third commodity is invented and in use as a public good, with the purchase of the good made by the Lindahl mechanism.

Production of country A

This part is exactly the same as the corresponding part in the previous sebsection.

Consumption of country A

We proceed to the demand side of country A. She is endowed with $L_{cA}$ and $K_{cA}$, exactly the same as in the private good case. All the agents in this paper: workers, capitalists, and entrepreneurs, have the same CES utility function, $u(y, x, z)$.

All the consumers maximize utility subject to income constraint:
\[
\text{max } u[y, x, z] \text{ s.t. } p_y y + p_x x + \theta_j p_z z = m_j \quad (j=L, K, 1, 2, 3) \tag{1}
\]

where \(m_j\) is income and \(\theta_j\) is the burden share of the household \(j\) \((j=L, K, 1, 2, 3)\) since the third commodity is procured by the Lindahl mechanism. Worker (household \(L\))’s income, \(m_L\), consists of initial endowment of labor, evaluated by the wage rate: \(w_LL_{cA}\). It is assumed that they supply \(L_{cA}\) for labor supply. Capitalist (household \(K\))’s income, \(m_K\), consists of initial endowment of capital, evaluated by the rental price of capital: \(w_KK_{cA}\). It is assumed that they supply \(K_{cA}\) for capital supply. Entrepreneur 1 (household 1)’s income, \(m_1\), consists of profit for the sector 1, \(\pi_{1A}\). Entrepreneur 2 (household 2)’s income, \(m_2\), consists of profit for the sector 2, \(\pi_{2A}\). Finally, entrepreneur 3 (household 3)’s income, \(m_3\), consists of profit for the sector 3, \(\pi_{3A}\).

Demand function of workers for commodity \(y\), \(y_L^D\), that for commodity \(x\), \(x_L^D\), that for commodity \(z\), \(z_L^D\), demand function of capitalists for commodity \(y\), \(y_K^D\), that for commodity \(x\), \(x_K^D\), that for commodity \(z\), \(z_K^D\), demand function of entrepreneur 1 for commodity \(y\), \(y_{E1}^D\), that for commodity \(x\), \(x_{E1}^D\), that for commodity \(z\), \(z_{E1}^D\), and demand function of entrepreneur 2 for commodity \(y\), \(y_{E2}^D\), that for commodity \(x\), \(x_{E2}^D\), that for commodity \(z\), \(z_{E2}^D\), are derived.

Country A’s demand for commodity \(y\), \(y_A^D\), is the sum of \(y_L^D\), \(y_K^D\), \(y_{E1}^D\), \(y_{E2}^D\), and \(y_{E3}^D\). Country A’s demand for commodity \(x\), \(x_A^D\), is the sum of \(x_L^D\), \(x_K^D\), \(x_{E1}^D\), \(x_{E2}^D\), \(x_{E3}^D\). Country A’s consumption of commodity \(z\) is determined by the Lindahl mechanism: \(z_A^D=z_L^D=z_K^D=z_{E1}^D=z_{E2}^D=z_{E3}^D\) for suitable selection of \(\theta_j\).

- **“Modified GE with Public good” and Gini Coefficients**

  General equilibrium for country A with innovation, “modified GE with public good”, \(\{p_y^{**}, p_x^{**}, p_z^{**}, w_K^{**}, \theta_L, \theta_K, \theta_1, \theta_2, \theta_3\}\) where \(\theta_j \geq 0\) and \(\theta_L + \theta_K + \theta_1 + \theta_2 + \theta_3 = 1\), is defined by

  \[
y_A^D = y_A^S, \quad (2)
  
x_A^D = x_A^S, \quad (3)
  
z_{AL}^D = z_{AK}^D = z_{AE1}^D = z_{AE2}^D = z_{AE3}^D \quad (4)
  
  L_{1A}^D + L_{2A}^D + L_{3A}^D = L_{cA}, \quad (5)
  
  K_{1A}^D + K_{2A}^D + K_{3A}^D = K_{cA}. \quad (6)
  
  \]

By the Newton method (or Walras-Lindahl mechanism), the “modified GE with public good” can be computed. The “modified GE with public good” incomes for 5 economic agents before and after the tax as the burden of consuming public good, are derived. Utilizing the “modified GE with public good” incomes for 5 agents, the Gini coefficient before the tax, \(\text{gini}_{11}\), and the Gini coefficient after the tax, \(\text{gini}_{12}\), are...
computed.

It is defined that if $\text{gini}_{A_0} < \text{gini}_{i_1}$, the income distribution before the tax on public good is more inequal when the new invention is public good and if $\text{gini}_{A_0} < \text{gini}_{i_2}$, the income distribution after the tax on public good is more inequal when the new invention is public good.

II: Simulations

The following Mathematica programing is provided for a simulation which computes the “probability” of the income distribution under “public-good invention case” before the tax being more unequal than the one under “private-good invention case”: i.e. the probability of $\text{gini}_{A_0} < \text{gini}_{i_1}$, as well as the “probability” of the income distribution under “public-good invention case” after the tax being more unequal than the one under “private-good invention case”: i.e. the probability of $\text{gini}_{A_0} < \text{gini}_{i_2}$.

Formally, first, 100 tuples of parameters are selected randomly where $a_i = m_i/n_i$, $b_i = s_i/t_i$, $a_i + b_i < 1$ (i = 1, 2, 3) for integers $m_i, n_i, s_i$, and $t_i$, selected randomly from the interval, [1,10], $a = u/v < 1$ for integers $u$ and $v$, selected randomly from the interval, [1, 10], and $L_{\text{CA}}$ and $K_{\text{CA}}$ are integers selected randomly from the interval, [1, 1000].

For each tuple, we compute “modified GE” with private good invention and “modified GE” with public good invention. The simulation, however, utilizes Newton method with the fixed initial values for the computation of “modified GE” with public good, so that we cannot always compute it depending on the initial values. Thus, we define the “probability” of the income distribution under “public-good invention case” before the tax being more unequal than the one under “private-good invention case” by the ratio (or percent) in the following way. The probability is the percent of cases in which $\text{gini}_{A_0} < \text{gini}_{i_1}$ holds among the successful Newton method application. In the same way, we define the “probability” of the income distribution under “public-good invention case” after the tax being more unequal than the one under “private-good invention case” by the ratio (or percent) in the following way. The probability is the percent of cases in which $\text{gini}_{A_0} < \text{gini}_{i_2}$ holds among the successful Newton method application.

\[
\text{In}[2]:= \text{m} = 100;
\]

\[
\text{In}[3]:= \text{data1} = \text{Table[}
\text{k11 := (g[x_] := \text{If}[x[[1]] < 1 \&\& x[[2]] < 1 \&\& x[[1]] + x[[2]] < 1, x,}
\text{]}
\]
f = \text{L}^a * K^b; \quad c1 = \{a \rightarrow a1, b \rightarrow b1, L \rightarrow L1, K \rightarrow K1\};
\text{f1 = f / . c1;}
c2 = \{a \rightarrow a2, b \rightarrow b2, L \rightarrow L2, K \rightarrow K2\}; f2 = f / . c2;
c3 = \{a \rightarrow a3, b \rightarrow b3, L \rightarrow L3, K \rightarrow K3\}; f3 = f / . c3;
\text{pi1 = py * f1 - wL * L1 - wK * K1;}
\text{sol1 =}
\text{Simplify[}
\text{Factor[Solve[}\{D[pi1, L1] = 0, D[pi1, K1] = 0\},
\{K1, L1\}][[1]]]];\text{demand$L1 = L1 / . sol1; demand$K1 = K1 / . sol1;}
\text{supply$A$y = Simplify[PowerExpand[f1 / . sol1]];}
\text{pi10 = FullSimplify[PowerExpand[pi1 / . sol1]];}
\text{pi2 = px * f2 - wL * L2 - wK * K2;}
\text{sol2 =}
\text{Simplify[}
\text{Factor[}
\text{PowerExpand[Solve[}\{D[pi2, L2] = 0, D[pi2, K2] = 0\},
\{K2, L2\}][[1]]]];\text{demand$L2 = L2 / . sol2; demand$K2 = K2 / . sol2;}
\text{supply$A$x = Simplify[PowerExpand[f2 / . sol2]];}
\text{pi20 = FullSimplify[PowerExpand[pi2 / . sol2]]];}
\[\pi_3 = \text{power} 3 - \text{w}_L \times \text{L}_3 - \text{w}_K \times \text{K}_3;\]
\[\text{sol3} =\]
\[\text{Simplify}\left[\text{Factor}\left[\text{PowerExpand}\left[\text{Solve}\left[\{\text{D}[\pi_3, \text{L}_3] = 0, \text{D}[\pi_3, \text{K}_3] = 0, \text{K}_3, \text{L}_3]\right]\right]\right]\right];\]
\[\text{demandL}_3 = \text{L}_3 / \text{sol3}; \text{demandK}_3 = \text{K}_3 / \text{sol3};\]
\[\text{supplyA}$z = \text{Simplify}\left[\text{PowerExpand}\left[\text{f}_3 / \text{sol3}\right]\right];\]
\[\text{pi}30 = \text{FullSimplify}\left[\text{PowerExpand}\left[\text{f}_3 / \text{sol3}\right]\right];\]
\[\text{u} = \left(x \times \text{a} + y \times \text{a} + z \times \text{a}\right)^{(1 / \text{a})};\]
\[\text{d}1 = \{\text{D}[u, x] / \text{D}[u, y] = \text{px} / \text{py}, \text{D}[u, x] / \text{D}[u, z] = \text{px} / \text{pz}, \]
\[\text{px} \times x + \text{py} \times y + \text{pz} \times z = m\};\]
\[\text{d}2 = \text{PowerExpand}\left[\text{Solve}\left[\text{d}1[[1]], y\right]\right][[1]];\]
\[\text{d}3 = \text{PowerExpand}\left[\text{Solve}\left[\text{d}1[[2]], z\right]\right][[1]];\]
\[\text{dx} = \text{FullSimplify}\left[\text{Solve}\left[\left(\text{d}1[[3]] / \text{d}2 / \text{d}3, x\right][[1]\right]\right];\]
\[\text{dy} = \text{Simplify}\left[\text{d}2 / \text{dx}\right]; \text{dz} = \text{d}3 / \text{dx};\]
\[\text{sol3A} = \{\text{dx}[[1]], \text{dy}[[1]], \text{dz}[[1]]\};\]
\[\text{demandL}_x = x / \text{sol3A} / \text{m} \rightarrow \text{w}_L \times \text{LeA};\]
\[\text{demandL}_y = y / \text{sol3A} / \text{m} \rightarrow \text{w}_L \times \text{LeA};\]
\[\text{demandL}_z = z / \text{sol3A} / \text{m} \rightarrow \text{w}_L \times \text{LeA};\]
\[\text{demandK}_x = x / \text{sol3A} / \text{m} \rightarrow \text{w}_K \times \text{KeA};\]
\[\text{demandK}_y = y / \text{sol3A} / \text{m} \rightarrow \text{w}_K \times \text{KeA};\]
\[\text{demandK}_z = z / \text{sol3A} / \text{m} \rightarrow \text{w}_K \times \text{KeA};\]
\[\text{demandE}_1 = x / \text{sol3A} / \text{m} \rightarrow \text{pi}10;\]
\[\text{demandE}_2 = x / \text{sol3A} / \text{m} \rightarrow \text{pi}20;\]
\[\text{demandE}_3 = x / \text{sol3A} / \text{m} \rightarrow \text{pi}30;\]
\[\text{demandE}_1 = y / \text{sol3A} / \text{m} \rightarrow \text{pi}10;\]
\[\text{demandE}_2 = y / \text{sol3A} / \text{m} \rightarrow \text{pi}20;\]
\[\text{demandE}_3 = y / \text{sol3A} / \text{m} \rightarrow \text{pi}30;\]
\[\text{demandE}_1 = z / \text{sol3A} / \text{m} \rightarrow \text{pi}10;\]
\[\text{demandE}_2 = z / \text{sol3A} / \text{m} \rightarrow \text{pi}20;\]
\[\text{demandE}_3 = z / \text{sol3A} / \text{m} \rightarrow \text{pi}30;\]
\{\text{demandL}_y, \text{demandL}_x, \text{demandL}_z, \text{demandK}_y, \}
\[\text{demandK}_x, \text{demandK}_z, \text{demandE}_1 y, \text{demandE}_1 x, \]
\[\text{demandE}_1 z, \text{demandE}_2 y, \text{demandE}_2 x, \text{demandE}_2 z, \]
\[\text{demandE}_3 y, \text{demandE}_3 x, \text{demandE}_3 z\};\]
\[\text{demandA}$y =\]
\[\text{Simplify}\left[\text{demandL}_y + \text{demandK}_y + \text{demandE}_1 y + \right.\]
\[\left.\text{demandE}_2 y + \text{demandE}_3 y\right];\]
\[\text{demandA}$x =\]
\[\text{Simplify}\left[\text{demandL}_x + \text{demandK}_x + \text{demandE}_1 x + \right.\]
\[\left.\text{demandE}_2 x + \text{demandE}_3 x\right];\]
demand$E2$x + demand$E3$x);
demand$A$z = 
Simplify[demand$L$z + demand$K$z + demand$E1$z +
demand$E2$z + demand$E3$z];
checkTA = {demand$A$x - supply$A$x, demand$A$y - supply$A$y,
demand$A$z - supply$A$z,
(demand$K1$ + demand$K2$ + demand$K3$) - KeA} /. data;
checkT = checkTA /. {px -> px[t], py -> py[t],
    pz -> pz[t], wK -> wK[t]};
solT1 =
NDSolve[{{D[px[t], t] == checkT[[1]],
    D[py[t], t] == checkT[[2]],
    D[pz[t], t] == checkT[[3]],
    D[wK[t], t] == checkT[[4]], px[0] == 1, py[0] == 1,
    pz[0] == 1, wK[0] == 1}, {px[t], py[t], pz[t], wK[t]},
    {t, 0, 100 000}, AccuracyGoal -> Infinity,
    PrecisionGoal -> 8][[1]]; 
solT2 = {px[t], py[t], pz[t], wK[t]} /. solT1 /. 
t -> 100 000;
sol6 = {px -> solT2[[1]], py -> solT2[[2]], pz -> solT2[[3]],
    wK -> solT2[[4]]};
checkTAA = checkTA /. data /. sol6;
income$A$LO = wL * LeA /. data;
N[income$A$K0 = wK * KeA /. sol6 /. data];
N[income$A$E10 = pi10 /. sol6 /. data];
N[income$A$E20 = pi20 /. sol6 /. data];
N[income$A$E30 = pi30 /. sol6 /. data];
giniA[y_] := Module[{z1, z2}, z1 = Sort[y];
    z2 = (y[[1]] + y[[2]] + y[[3]] + y[[4]] + y[[5]]) / 5;
    1 + (1/5) -
    2 (z1[[5]] + 2 * z1[[4]] + 3 * z1[[3]] + 4 * z1[[2]] +
    5 * z1[[1]]) / (5^2 * z2)];
giniA0 =
giniA[{income$A$LO, income$A$K0, income$A$E10, income$A$E20, income$A$E30}];
sol3AA = PowerExpand[sol3A /. {pz -> tj * pz}];
demand$L$xA = x /. sol3AA /. {m -> wL * LeA, tj -> tL};
demand$L$yA = y /. sol3AA /. {m -> wL * LeA, tj -> tL};
demand$L$zA = z /. sol3AA /. {m -> wL * LeA, tj -> tL};
demand$K$xA = x /. sol3AA /. {m -> wK * KeA, tj -> tK};
demand$K$yA = y /. sol3AA /. {m -> wK * KeA, tj -> tK};
demand$K$zA = z /. sol3AA /. {m -> wK * KeA, tj -> tK};
demand$E1$xA = x /. sol3AA /. {m -> pi10, tj -> t1};
demand$E2$xA = x /. sol3AA /. {m -> pi20, tj -> t2};
demand$E3$xA = x /. sol3AA /. {m -> pi30, tj -> t3};
demand$E1$yA = y /. sol3AA /. {m -> pi10, tj -> t1};
demand$E2$yA = y /. sol3AA /. {m -> pi20, tj -> t2};
demand$E3$yA = y /. sol3AA /. {m -> pi30, tj -> t3};
demand$E1$zA = z /. sol3AA /. {m -> pi10, tj -> t1};
demand$E2$zA = z /. sol3AA /. {m -> pi20, tj -> t2};
demand$E3$zA = z /. sol3AA /. {m -> pi30, tj -> t3};
demand$A$yA = Simplify[demand$L$yA + demand$K$yA + demand$E1$yA +
demand$E2$yA + demand$E3$yA];
demand$A$xA = Simplify[demand$L$xA + demand$K$xA + demand$E1$xA +
demand$E2$xA + demand$E3$xA];
checkTTT = {demand$A$xA - supply$A$x, demand$A$yA - supply$A$y, 
(demand$L$zA + demand$K$zA + demand$E1$zA +
demand$E2$zA + demand$E3$zA) / 5 - supply$A$z,
demand$L$zA - 
(demand$L$zA + demand$K$zA + demand$E1$zA +
demand$E2$zA + demand$E3$zA) / 5,
demand$K$zA -
(demand$L$zA + demand$K$zA + demand$E1$zA +
demand$E2$zA + demand$E3$zA) / 5,
demand$E1$zA -
(demand$L$zA + demand$K$zA + demand$E1$zA +
demand$E2$zA + demand$E3$zA) / 5,
demand$E2$zA -
(demand$L$zA + demand$K$zA + demand$E1$zA +
demand$E2$zA + demand$E3$zA) / 5,
demand$K$zA - 
(demand$L$zA + demand$K$zA + demand$E1$zA +
demand$E2$zA + demand$E3$zA - KeA) /. data;
solT1A = FindRoot[{checkTTT[[1]] == 0, checkTTT[[2]] == 0, 
checkTTT[[3]] == 0, checkTTT[[4]] == 0, 
checkTTT[[5]] == 0, checkTTT[[6]] == 0, 
checkTTT[[7]] == 0, checkTTT[[8]] == 0}, 
{px, (px /. sol6)}, {py, (py /. sol6)}, 
{pz, (pz /. sol6)}, {tL, 1/5}, {tK, 1/5}, 
{t1, 1/5}, {t2, 1/5}, {wK, (wK /. sol6)}, 
AccuracyGoal -> Infinity, PrecisionGoal -> 10,
MaxIterations \rightarrow 10000;
checkT = checkTTT /. {py \rightarrow py[t], px \rightarrow px[t], pz \rightarrow pz[t],
tL \rightarrow tL[t], tK \rightarrow tK[t], t1 \rightarrow t1[t], t2 \rightarrow t2[t],
wK \rightarrow wK[t]};
checkTTTA = checkTTT /. solT1A;
sol100 = solT1A;
{income$A$L0A = wL \times LeA /. data /. sol100 /. wL \rightarrow 1,
income$A$L0B =
  (wL \times LeA - tj \times pz \times z) /. {tj \rightarrow tL, z \rightarrow demand$L$z} /. data /. sol100 /. wL \rightarrow 1};
{income$A$K0A = (wK \times KeA) /. data /. sol100 /. wL \rightarrow 1,
income$A$K0B =
  (wK \times KeA - tj \times pz \times z) /. {tj \rightarrow tK, z \rightarrow demand$K$z} /. data /. sol100 /. wL \rightarrow 1};
{income$A$E10A =
  (pi10) /. {tj \rightarrow t1, z \rightarrow demand$E1$z} /. data /. sol100 /. wL \rightarrow 1,
income$A$E10B =
  (pi10 - tj \times pz \times z) /. {tj \rightarrow t1, z \rightarrow demand$E1$z} /. data /. sol100 /. wL \rightarrow 1};
{income$A$E20A =
  (pi20) /. {tj \rightarrow t2, z \rightarrow demand$E2$z} /. data /. sol100 /. wL \rightarrow 1,
income$A$E20B =
  (pi20 - tj \times pz \times z) /. {tj \rightarrow t2, z \rightarrow demand$E2$z} /. data /. sol100 /. wL \rightarrow 1};
{income$A$E30A =
  (pi30) /. {tj \rightarrow t3, z \rightarrow demand$E3$z} /.
    {t3 \rightarrow 1 - t1 - t2 - tL - tK} /. data /. sol100 /. wL \rightarrow 1, income$A$E30B =
  (pi30 - tj \times pz \times z) /. {tj \rightarrow t3, z \rightarrow demand$E3$z} /. data /. sol100 /. wL \rightarrow 1};
gini11 = giniA[{income$A$L0A, income$A$K0A, income$A$E10A, income$A$E20A, income$A$E30A}];
gini12 = giniA[{income$A$L0B, income$A$K0B, income$A$E10B, income$A$E20B, income$A$E30B}];
{checkTAA, checkTTTTA, sol100, giniA0, gini11, gini12}, {m}];
In[4]:= data2 = Select[data1, 
   Abs[#1 [1, 1]] < 10^-8 && Abs[#1 [1, 2]] < 10^-8 && 
   Abs[#1 [1, 3]] < 10^-8 && 
   Abs[#1 [1, 4]] < 10^-8 && 
   Abs[#1 [2, 1]] < 10^-8 && 
   Abs[#1 [2, 2]] < 10^-8 && 
   Abs[#1 [2, 3]] < 10^-8 && 
   Abs[#1 [2, 4]] < 10^-8 && 
   Abs[#1 [2, 5]] < 10^-8 && 
   Abs[#1 [2, 6]] < 10^-8 && 
   Abs[#1 [2, 7]] < 10^-8 && 
   Abs[#1 [2, 8]] < 10^-8 && (t1 / . #1 [[3]]) >= 0 && 
   (t2 / . #1 [[3]]) >= 0 && (tL / . #1 [[3]]) >= 0 && 
   (tK / . #1 [[3]]) <= 1 && ];

In[5]:= Length[data2]
Out[5]= 76

In[6]:= {Length[Select[data2, #4 < #5 && ]] / Length[data2], 
   Length[Select[data2, #4 < #6 && ]] / Length[data2]}

In[7]:= N[%]
Out[7]= {0.605263, 0.289474}

In this simulation with 100 tuples, there are 76 successful Newton method applications. Among them, 46 cases show that \text{gini}_{A0}<\text{gini}_{11}, so that the probability of \text{gini}_{A0}<\text{gini}_{11} is 0.605263, while 22 cases show that \text{gini}_{A0}<\text{gini}_{12}, so that the probability of \text{gini}_{A0}<\text{gini}_{12} is 0.289474. The same simulation was repeated 50 times with the following result.

\begin{verbatim}
sdata[9] = \left\{ \frac{22}{37}, \frac{25}{74} \right\}; sdata[10] = \left\{ \frac{17}{35}, \frac{9}{35} \right\};
sdata[11] = \left\{ \frac{19}{40}, \frac{23}{80} \right\}; sdata[12] = \left\{ \frac{1}{2}, \frac{25}{82} \right\};
sdata[13] = \left\{ \frac{41}{84}, \frac{29}{84} \right\}; sdata[14] = \left\{ \frac{1}{2}, \frac{11}{39} \right\};
sdata[15] = \left\{ \frac{21}{38}, \frac{21}{76} \right\}; sdata[16] = \left\{ \frac{37}{71}, \frac{21}{71} \right\};
sdata[17] = \left\{ \frac{11}{25}, \frac{16}{75} \right\}; sdata[18] = \left\{ \frac{1}{2}, \frac{20}{81} \right\};
sdata[19] = \left\{ \frac{38}{81}, \frac{17}{81} \right\}; sdata[20] = \left\{ \frac{42}{73}, \frac{19}{73} \right\};
sdata[21] = \left\{ \frac{1}{2}, \frac{13}{40} \right\}; sdata[22] = \left\{ \frac{43}{79}, \frac{22}{79} \right\};
sdata[23] = \left\{ \frac{11}{25}, \frac{17}{75} \right\}; sdata[24] = \left\{ \frac{19}{40}, \frac{19}{80} \right\};
sdata[25] = \left\{ \frac{7}{13}, \frac{4}{13} \right\}; sdata[26] = \left\{ \frac{45}{79}, \frac{22}{79} \right\};
sdata[27] = \left\{ \frac{45}{83}, \frac{23}{83} \right\}; sdata[28] = \left\{ \frac{29}{76}, \frac{7}{38} \right\};
sdata[29] = \left\{ \frac{16}{41}, \frac{17}{82} \right\}; sdata[30] = \left\{ \frac{21}{40}, \frac{21}{80} \right\};
sdata[31] = \left\{ \frac{16}{41}, \frac{7}{41} \right\}; sdata[32] = \left\{ \frac{44}{79}, \frac{28}{79} \right\};
sdata[33] = \left\{ \frac{14}{25}, \frac{7}{25} \right\}; sdata[34] = \left\{ \frac{10}{19}, \frac{11}{38} \right\};
sdata[35] = \left\{ \frac{46}{85}, \frac{23}{85} \right\}; sdata[36] = \left\{ \frac{22}{41}, \frac{13}{41} \right\};
sdata[37] = \left\{ \frac{41}{79}, \frac{22}{79} \right\}; sdata[38] = \left\{ \frac{7}{13}, \frac{7}{26} \right\};
sdata[39] = \left\{ \frac{16}{29}, \frac{22}{87} \right\}; sdata[40] = \left\{ \frac{37}{69}, \frac{6}{23} \right\};
sdata[41] = \left\{ \frac{22}{35}, \frac{12}{35} \right\}; sdata[42] = \left\{ \frac{48}{79}, \frac{18}{79} \right\};
sdata[43] = \left\{ \frac{38}{79}, \frac{23}{79} \right\}; sdata[44] = \left\{ \frac{23}{41}, \frac{10}{41} \right\};
sdata[45] = \left\{ \frac{56}{83}, \frac{32}{83} \right\}; sdata[46] = \left\{ \frac{41}{78}, \frac{7}{26} \right\};
sdata[47] = \left\{ \frac{15}{26}, \frac{9}{26} \right\}; sdata[48] = \left\{ \frac{49}{90}, \frac{14}{45} \right\};
\end{verbatim}
The mean of the probability of $\text{gini}_{A0} < \text{gini}_{11}$ is 0.514488. This, however, cannot assert that the probability of $\text{gini}_{A0} < \text{gini}_{11}$ is greater than 0.5, as is clear from the following collection of each result for simulation with 100 tuples. It may be safe to conclude that the probability of $\text{gini}_{A0} < \text{gini}_{11}$ is 0.5.

Meanwhile, the mean of the probability of $\text{gini}_{A0} < \text{gini}_{12}$ is 0.276258. We may safely conclude that the probability of $\text{gini}_{A0} < \text{gini}_{12}$ is lower than 0.5, as clear from the following collection of each result for simulation with 100 tuples.
Conclusion

Prominent economists such as P. Krugman and L. Summers argued that excessive income distribution inequality in the US may well lead to the collapse of “social contract”. The expansion of income distribution inequality, however, has been noticed worldwide. The purpose of the present paper is to examine the relation between the innovation and income distribution inequality. The innovation in this paper is defined as the new commodity invention, following J.A.Schumpeter. This examination is of some interest theoretically especially when we consider the economic history in the “peace dividends” era. After the collapse of communism the US president Bill Clinton could shift the government fund from the military purpose to the non-military purpose, leading to the promotion of ICTs: the emergence of “new economy” in the 1990s, which disappeared after the September 11 terrorism. It is interesting, in this way, to examine how the income distribution inequality, Gini coefficient, differs between the case in which the government fund is used for the promotion of innovating military good and a new military good is invented and used as a public good via Lindahl-type taxation and the one in which the new commodity is invented as a private good. Utilizing general equilibrium simulation, we show that the Gini coefficient is concluded as equal between the two cases when it is computed for the incomes before taxation on the public good, while it is definitely greater for the private good case when it is computed for the incomes after taxation on the public good.

References


Fukiharu, T. [2009], “Information and Communication Technologies and the Income Distribution: A Simulation through Inequality Measures”, Anderssen, R.S., R.D.

http://www.cc.aoyama.ac.jp/~fukito/IndexII.htm

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